Ciphering Fall Classic 2003

Name_____

School_____

Grade_____

Team #_____

1) The sum of the digits of James' positive two-digit number is 9. If the number is not a multiple of 6, is not a perfect square, and is not a perfect cube, list in order from smallest to largest all possible solutions.

ANSWER_____ 45, 63 in that order

2) For how many positive integers m $\leq$ 100 is (m - 5)(m - 23)(m – 68) positive?

ANSWER_____49

3) If Mary bought 5 apples the first week, 6 apples the second week, 7 apples the third week and this pattern continued for 20 weeks, how many total apples did Mary purchase at the end of the 20 weeks?

ANSWER_____ 290

4) How many positive three-digit numbers are there for which the tens digit exceeds the sum of the other two digits?

ANSWER_____ 120

5) Jill would like to design a rectangular prism with a volume of 32 cubic feet such that one edge is twice as long as another and each dimension is an integer. How many unique prisms could she make?

ANSWER_____ 3

6) A lock has 5 buttons numbered 1-5. The lock is opened by pushing two buttons simultaneously and then pushing one button alone. How many different combinations are possible?

ANSWER_____ 50

7) Christina is building a circular corral for her dog Spot. To form the corral he pounds 20 posts equally spaced around the circumference. Each post contributes 6 inches to the circumference and the fencing between adjacent posts contributes 8 feet to the circumference. To the nearest integer, what is the number of square feet in the area of the corral? Use $\frac{22}{7}$ for π.

ANSWER_____ 2,299

8) If a, b and c each represent a different integer 0-9, what is the value of c?

$$
\begin{array}{r}
aaaa \\
bbbb \\
+ \quad cccc \\
\hline
baaac
\end{array}
$$

ANSWER_____ 8

9) Selina's age on her birthday in 1993 is equal to the sum of the digits of her birth year. How old is she?

ANSWER_____ 20

10) What is the value of the largest mean among the following sets of whole numbers?

> A: all the multiples of 3 between 1 and 50,
> B: all the multiples of 4 between 1 and 50,
> C: all the multiples of 6 between 1 and 50,
> D: all the multiples of 7 between 1 and 50,
> E: all the multiples of 9 between 1 and 50.

ANSWER_____ 28

11)   On a true/false test of 100 items, every question that is a multiple of 4 is true, and all others are false. If a student marks every item that is a multiple of 3 false and all others true, how many of the 100 items will be correctly answered?

ANSWER_____ 42

12)   Step 1:   Solve for x

$$3x - 3(x - 3) - 3(x - 3) - (3 - x) - 3(3 - x) = 4(x - 4) - 4(4 - x) - 4$$

Step 2:   Insert the value for x into the following and find y

$$y = x^2 - x^3 + x^4$$

Step 3:   Find the sum of the digits of y

ANSWER_____ 9

13) On a number line Point A has the coordinate –8 and Point B has the coordinate 13. What are the coordinates of all points on the line whose distance from Point A is two-thirds of its distance from Point B?

ANSWER_____ $\dfrac{2}{5}$ and -50

14) If you use 999 digits to write page numbers consecutively starting with 1, how many page numbers could you write?

ANSWER_____ 369

15) What is the unit's digit of the sum of the following?

1! + 2! + 3! + 4! + …………..14! + 15!

.

ANSWER_____ 3

16) What is the smallest positive integer that cannot be expressed as the sum of eight or fewer cubes of positive integers?

ANSWER_____ 23

17)   What is the largest 7-digit number that contains each of the digits 1-7 and
      has the property that the sum of any two consecutive digits is a prime number?

ANSWER_____ 7652341

18)   Of the following which has the greatest volume in cubic centimeters, the
      cube, the prism, the pyramid or the sphere?

      A cube with edge = 12cm
      A rectangular prism with length = 11cm, width = 12cm and height = 13cm
      A square pyramid with the area of the base 160cm^2, and height of 30cm
      Or a sphere with radius 7cm (Use 3.14 for π)

ANSWER_____ the cube

19) In a tennis tournament, n women and 2n men play, and each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number of matches won by men is 7:5, then n equals?

ANSWER_____ 3

20)   A jar of marbles has 40 blue marbles and 24 red marbles and 36 orange marbles. What is the probability that I randomly select 3 marbles from the jar and they are all of different colors? Express your answer as a reduced fraction.

ANSWER_____ $\dfrac{576}{2695}$