Ciphering Fall Classic 2004

Name_________________________________

School________________________________

Grade________________________________

Team #________________

1) The digits 1, 3, 4, 6, 8, and 9 are each used exactly once to make three two-digit positive primes. What is the sum of these primes?

ANSWER______________________ 193

2) E.T. wants to phone home using a pay phone. Problem is that he remembers his 7-digit home number except for the last digit. To add insult to injury, he only has enough money for two attempts. What is the probability that E.T. will successfully phone home before running out of money? Express your answer as a common fraction.

ANSWER______________________ 1/5

3) What is the smallest positive integer N such that the value of $7 + 30 \times N$ is not a prime number?

ANSWER______________________ 6

4) Among all right triangles with sides of integer lengths less than 50 with areas numerically equal to their perimeters, find the ratio of the triangle with the greatest area to the triangle of smallest area. Express your answer as a common fraction.

ANSWER______________________ 5/4

5) How many positive integers less than 50 are there with the property that all the positive integers less than or equal to the square root of the number divide evenly into the number?

ANSWER______________________ 8

6) It can be shown that 27 is the largest integer equal to the sum of the digits of its cube (note that $27^3 = 19683$ and 27 = 1 + 9 + 6 + 8 + 3). What is the second largest integer with this property?

ANSWER________________ 26

7) What is the smallest perfect square contained in the solution set of the following inequality?

$$\frac{x+34}{8}+\frac{2x-1}{10}>\frac{x}{4}+21$$

ANSWER______________________ 225

8) James has a spinner that is labeled 1-8, and Dana has a spinner labeled 1-15. On each spinner, each number has an equal probability of occurring. If each spinner is spun, what is the probability that the product of the spins is greater than 50? Express your answer as a common fraction.

ANSWER______________________ 4/15

9) Given the following numbers:

- A = The sum of the first 50 positive integers
- B = The sum of the first 50 odd integers
- C = The sum of the first 50 even integers

Find the value of A – B + C.

ANSWER______________________ 1325

10) Square ABCD is inscribed inside a circle with radius of length 10. If P is a point on the circle, find the value of $(PB)^2 + (PD)^2$.

ANSWER______________________ 400

11) A and B are positive integers such that $A^3 + B^2 = 850$, and $A^2 - B^3 = -1250$
What is $(A + B)^2$?

ANSWER_____________________ 400

12) In a ten-mile race, the winner beat second-place by two miles and third-place by four miles. Assuming the runners maintained a constant speed throughout the race, by how many miles does the second-place beat third-place? Express your answer as a common fraction.

ANSWER______________________ 5/2

13) Patrick wants *you* to guess his favorite integer, which is less than 1,000. The only hint he gives is that the number is 1 more than a perfect square and 1 less than a perfect cube. What is Patrick's favorite number?

ANSWER______________________ 26

14) Let a, b, c, and d be distinct numbers from the set {1, 2, 3, 4}. How many ways can $(a-1)(b-2)(c-3)(d-4)$ equal 0?

ANSWER______________________ 15

15) Solve for the *larger* value of *x*: $5^x + \frac{125}{5^x} = 30$

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ANSWER_____________________ 2

16) Given the following:

- A = 10000^{100}
- B = 2^{10000}
- C = 1000^{1000}
- D = 5^{4000}
- E = 3^{2000}

Arrange the above numbers from *smallest to largest* using the corresponding letter equivalent.

ANSWER______________________ A, E, D, C, B in that order

17) If x and y are positive integers and $\frac{x!}{y!} = 30$, find the mean of all possible values for x.

ANSWER______________________ 18

18) Let P be a point inside rectangle ABCD such that PA = 15, PB = 7, and PC = 20. What is the length of PD?

ANSWER______________________ 24

19) A rather strange hourglass is made from two identical right circular cones by gluing them at their vertices. In the beginning, the upper cone is filled to capacity with liquid and the lower one is empty. The hourglass is then activated and the liquid flows at a constant rate from the upper cone to the lower cone. It takes exactly one hour to empty the upper cone of liquid. How long, in hours, does it take for the depth of liquid in the lower cone to be half the depth of liquid in the upper cone? Express your answer as a common fraction.

ANSWER______________________ 19/27

20) Suppose the scoring system in football is changed so that field goals are worth 2 points and touchdowns are worth 5 points. How many ways are there to score 100 points? Only consider the number of field goals and touchdowns, not the order in which they occurred.

ANSWER______________________ 11