

# Team Test

## Fall Classic 2004

- 1) Which of the following numbers is divisible by 7?  
A) 392,161   B) 67,072   C) 262,656   D) 117,585   E) NOTA
- 2) What is the area of a triangle with side lengths 12, 17, and 25?  
A) 88   B) 90   C) 92   D) 94   E) NOTA
- 3) What is the sum of the two prime factors of  $390629 = 625^2 + 4$ ?  
A) 1,252   B) 1,254   C) 1,256   D) 1,258   E) NOTA
- 4) How many 9's appear as digits in the decimal expansion of  $9999998999999^2$ ?  
A) 9   B) 10   C) 11   D) 12   E) NOTA
- 5) The sum of the square of a number and the square of its reciprocal is 14. Which of the following could equal the sum of the number and its reciprocal?  
A) 16   B) 9   C) 4   D) 1   E) NOTA
- 6) After finding the average of 2,004 exam scores for his Introductory Calculus course, Professor Perkins accidentally included the average with the 2,004 scores and found the average of the 2,005 numbers. What is the ratio of the average of the 2,004 numbers to the average of the 2,005 numbers?  
A)  $\frac{3}{4}$    B)  $\frac{5}{4}$    C)  $\frac{2005}{2004}$    D) 1   E) NOTA
- 7) Find the sum of all positive integers  $n$  such that  $n^3 + 100$  is a multiple of  $n + 10$ .  
A) 1,218   B) 2,551   C) 5,625   D) 9,000   E) NOTA

8) A company run by a billionaire CEO has three different job openings, and the CEO is considering giving them to three applicants (one job per applicant) from a group of four entrepreneurs and six close relatives. Now in order to prevent any claims of nepotism, the CEO's advisors told him that at least two entrepreneurs must get a job. How many ways can the job assignments be done?

- A) 240      B) 256      C) 272      D) 288      E) NOTA

9) An urn contains a certain number of black and white marbles, and no marbles of other colors. If two marbles are randomly taken out of the urn, the probability that they are both white is  $\frac{1}{3}$ . However, if three marbles are taken out, the probability of them being all white decreases to  $\frac{1}{6}$ . How many *black* marbles are in this urn?

- A) 4      B) 5      C) 6      D) 7      E) NOTA

10) Four friends (Curtis, Tamara, David, and Tiffany) are sitting around a circular table talking about their respective sports of expertise (tennis, golf, swimming, or jogging). The following information is known:

- Curtis is sitting across from the tennis player.
- David is sitting across from the golfer.
- Tamara is sitting on Tiffany's left.
- The jogger is sitting on the swimmer's right.
- There is a boy sitting on Curtis' right.

Who is the golfer among the four?

- A) Tamara    B) Tiffany    C) Curtis    D) David    E) NOTA

11) Notice that 26, 39, and 201 can be written as a sum of consecutive integers (because  $26 = 5 + 6 + 7 + 8$ ,  $39 = 12 + 13 + 14$ , and  $201 = 100 + 101$ ). Which of the following *cannot* be written as a sum of consecutive integers?

- A) 2,005      B) 3,125      C) 4,096      D) 8,128      E) NOTA

$$x + \sqrt{yz} = 66$$

- 12) Given that  $y - \sqrt{xz} = 6$  for real numbers  $x$ ,  $y$ , and  $z$ , find the value of

$$z + \sqrt{xy} = 42$$

$x + y + z$ .

- A) 82      B) 83      C) 84      D) 85      E) NOTA

- 13) In a recent math competition, the attendants were asked the question “Do you like solving math problems?” with “Yes” or “No” as the only possible answers. An equal number of boys and girls responded, and 60% of those saying “Yes” were boys and 80% of those saying “No” were girls. What percentage of boys answered “No” to the question?

- A) 10%      B) 25%      C) 30%      D) 40%      E) NOTA

- 14) How many five-digit positive integers have the following properties?

- All the digits are odd.
- The positive difference between any two adjacent digits is 2.

- A) 40      B) 42      C) 44      D) 46      E) NOTA

- 15) A ten-centimeter wide rectangular strip with infinite length is folded exactly once. Find, in square centimeters, the smallest possible area of the region where the strip covers itself.

- A) 5      B) 15      C) 25      D) 50      E) NOTA

- 16) A set of integers has mean 22, mode 32, and median  $x$ , all of which belong to the set. The smallest number in this set is 10. If  $x$  is replaced by  $x + 10$ , the mean and median of the set would change to 24 and  $x + 10$ , respectively. On the other hand, if  $x$  is replaced by  $x - 8$ , the median would change to  $x - 4$ . Find the value of  $x$ .

- A) 14      B) 16      C) 18      D) 20      E) NOTA

17) Bob has a rather thick deck of 2,004 cards numbered from 1 to 2,004. Using these cards, he plays a game with his friend Jay as follows: first, the deck is shuffled and 1,002 cards are each dealt to Bob and Jay. Beginning with Jay, the two friends take turns discarding one of their cards and calling out its number. The game ends when the sum of the numbers on the discarded cards is divisible by 2005, and the winner is the last person to discard. Assuming that both Bob and Jay are playing optimally, find the probability Jay wins.

- A)  $\frac{1}{2}$       B)  $\frac{3}{5}$       C)  $\frac{2}{3}$       D) 1      E) NOTA

18) Container A has six liters of pure acid while container B has six liters of pure water. An empty bottle is filled with acid from container A and then completely emptied into container B. After stirring the solution evenly, the bottle is filled with this mixture from B and emptied into A; the result is that container A now contains four times as much acid as water. Assuming there were no spills incurred throughout the process, what is the capacity in liters of the bottle?

- A) 1      B)  $\frac{5}{4}$       C)  $\frac{3}{2}$       D)  $\frac{2}{5}$       E) NOTA

19) How many positive integers have the following properties?

- Has four digits.
- All the digits are odd.
- A multiple of 5.
- One-fifth of the number also has four digits.

- A) 28      B) 27      C) 26      D) 25      E) NOTA

20) Triangle  $ABC$  is a  $45^\circ$ - $45^\circ$ - $90^\circ$  right triangle with hypotenuse  $AB$  of length  $4\sqrt{2}$ . Let  $P$  be the midpoint of  $BC$  and let  $Q$  and  $R$  be points on  $AC$  and  $AB$ , respectively. What is the smallest possible perimeter for triangle  $PQR$ ?

- A)  $\frac{7\sqrt{3}}{4}$       B)  $\sqrt{40}$       C)  $4 + \sqrt{2}$       D) 8      E) NOTA

- 1) A
- 2) B
- 3) B
- 4) C
- 5) C
- 6) D
- 7) B
- 8) A
- 9) A
- 10) B
- 11) C
- 12) C
- 13) A
- 14) B
- 15) C
- 16) D
- 17) E (0)
- 18) C
- 19) B
- 20) B