2005 Fall Classic Team Test

1) Write 63522_7 as a base-5 numeral.

A) 1000241_5 B) 521072_5 C) 15696_5 D) 9100_5 E) NOTA

2) Evaluate $\left(\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-c)(b-a)}+\frac{1}{c(c-a)(c-b)}\right)^2$ when $a=1/2$, $b=1/3$, and $c=1/4$.

A) $\frac{25}{16}$ B) 576 C) 144 D) 0 E) NOTA

3) Let $S=1^2+2^2+3^2+\cdots+299^2+300^2$, the sum of the squares of the first three hundred counting numbers. What is the sum of the digits of *S*?

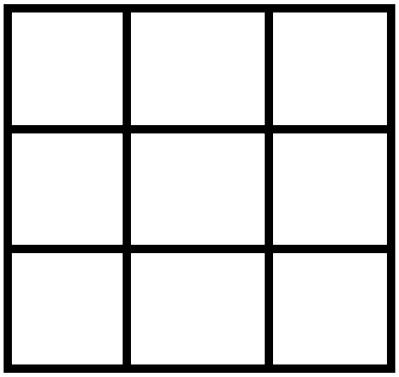
A) 29 B) 27 C) 25 D) 23 E) NOTA

4) Find the area of a triangle with side lengths 16, 25, and 39. All side lengths are in meters.

A) 100 m² B) 110 m² C) 120 m² D) 130 m² E) NOTA

5) Each of the nine cells on the right is filled with a distinct positive integer such that the numbers in each row, column, and diagonal have the same product. What is the sum of the digits of the smallest such product?

A) 6 B) 7 C) 8 D) 9 E) NOTA



6) Alyssa participated in rock climbing competitions as a hobby. She did this for seventeen years, and throughout those years she earned a total of 72 million dollars. She was paid *x* million dollars a year (*x* being an integer) and received an extra one million dollars every time she qualified for the X-Games. During her years in competition she qualified for the X-Games *n* times. Find the sum of the cubes of all positive integers less than *n*.

A) 225 B) 100 C) 36 D) 9 E) NOTA

7) In the expression below, each letter is replaced with a distinct element from the set of the five smallest counting numbers:

$$(A + B^C - D)E$$

What is the largest possible value this expression can take?

A) 2,044 B) 3,075 C) 4,072 D) 5,100 E) NOTA

8) At the start of the day, Sarah has 6 gold coins in each of her two pockets. As the day goes on, she randomly reaches into one of her pockets without looking, takes out a coin (each pocket is equally likely to be chosen), and donates it to a worthy cause. Sometime in the day, she reaches into a pocket and finds it empty. What is the probability the other pocket contains 3 coins?

A) $\frac{21}{128}$ B) $\frac{7}{64}$ C) $\frac{105}{512}$ D) $\frac{55}{1,024}$ E) NOTA

9) A *median* of a triangle is a line segment from a vertex to the midpoint of the side opposite that vertex. Triangle ABC has AB = 25, BC = 24, and AC = 7. The lengths of the sides of triangle DEF are the same as the lengths of the medians of ABC. What is the ratio of the area of DEF to ABC?

A) $\frac{3}{4}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{5}$ E) NOTA

10) If Richard sets his alarm clock, he wakes up at 6am the next day without fail. Otherwise, he wakes up at 6am 30% of the time and 8am 70% of the time. If Richard wakes up at 6am, he has a 50% chance of setting his alarm clock for the next day. If Richard wakes up at 8am he has a 80% chance of setting his alarm clock for the next day. If Richard wakes up at 6am today, what is the chance he wakes up at 6am two days from today?

A) 20.25% B) 53.13% C) 35.00% D) 72.35% E) NOTA

11) The product of two numbers is 9 less than the square of one of the numbers and is 7 less than the square of the other number. Find the product of the two numbers.

A) –4 B) $\frac{1}{2}$ C) $-\frac{63}{16}$ D) 4 E) NOTA

12) Just for fun, Victoria decides to add up all the page numbers in her favorite fashion book, which has n pages starting at 1 (n is a positive integer). However, she accidentally adds a particular page number twice, leaving her with an incorrect total of 2005. What is the sum of the positive integral factors of the page number she counted twice?

A) 216 B) 168 C) 124 D) 98 E) NOTA

13) The population of the Village of the Hidden Leaves was originally a perfect square. Later, with the birth of a hundred babies, the population became one more than a perfect square. After the birth of a hundred more babies, the village population was once again a perfect square. What is the original population? Assume there were no deaths or anything that would decrease the population.

A) 1,665 B) 2,401 C) 2,926 D) 3,870 E) NOTA

14) A regular tetrahedron is inscribed in a right circular cone of radius 1 and height 2 such that one vertex of the tetrahedron coincides with the center of the cone's base and the face opposite this vertex is perpendicular to the axis of the cone. What is the volume of the tetrahedron?

A) $5\sqrt{6}-7\sqrt{3}$ B) $9\sqrt{3}-6\sqrt{6}$ C) $3\sqrt{6}-4\sqrt{3}$ D) $18\sqrt{3}-12\sqrt{6}$ E) NOTA

15) Kristina walks into her math class one morning and finds the first one hundred positive integers written on the board in some order. There was still some time before class, so she takes two numbers x and y from the board and replaces them with $x + y + 2$. Amused, she keeps doing this until there is only one number left on the board. What is this number?

A) 5,248 B) 5,429 C) 5,250 D) 5,251 E) NOTA

16) A *cryptarithm* is an arithmetic problem where each distinct digit has been replaced with a distinct letter, and the goal is to figure out what the digits are. A famous example of a cryptarithm is one that college students might be able to relate to:

$$\begin{array}{r} \text{SEND} \\ +\text{MORE} \\ \hline \text{MONEY} \end{array}$$

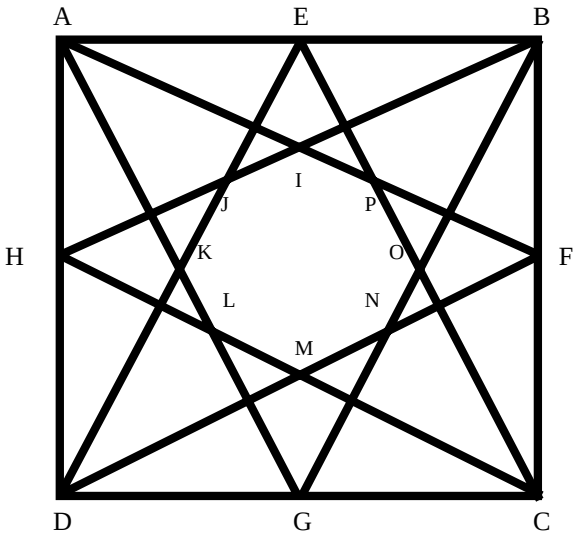
High school seniors, however, might feel more strongly about the following puzzle:

$$\begin{array}{r} \text{SAT} \\ +\text{ACT} \\ \hline \text{PAIN} \end{array}$$

If we want the smallest possible value for PAIN, what is the value of S + A?

A) 10 B) 11 C) 12 D) 13 E) NOTA

17) In the figure to the right, points E, F, G, and H are the midpoints of the sides of square ABCD. What is the ratio of the area of octagon IJKLMNOP to the area of ABCD? (Figure is ***not*** drawn to scale)



A) $\frac{5}{24}$ B) $\frac{1}{6}$ C) $\frac{3}{16}$ D) $\frac{1}{5}$ E) NOTA

18) The sum of the squares of the first and fourth terms of an arithmetic sequence is 1,205. The sum of the squares of the second and third terms is 881. What is the product of all four terms?

A) 132,860 B) 112,385 C) 95,200 D) 75,384 E) NOTA

19) Let ABCD be a convex quadrilateral where the lengths of all the sides and diagonals are rational numbers. If P is the intersection of AC and BD, which of the following lengths must also be rational?

I. AP
II. BP
III. CP
IV. DP

A) I only B) II and III only C) I and III only D) IV only E) NOTA

20) It can be shown that the $50{,}000^{th}$ prime is 611,953. What is the value of the $50{,}001^{st}$ prime?

A) 612,249 B) 612,501 C) 612,733 D) 612,861 E) NOTA