

# Ciphering Fall Classic 2006

## QUESTIONS

**Name** \_\_\_\_\_

**School** \_\_\_\_\_

**Grade** \_\_\_\_\_

**ID #** \_\_\_\_\_

1) Find  $A + B + C + D$  given that: (express your answer as a decimal)

$$A = (1.1)^2 \quad B = \frac{3}{5} \text{ of } .8 \quad C = 30\% \text{ of } .9 \quad D = .1 + .2 \times .3$$

ANSWER \_\_\_\_\_ 2.12

2) Two numbers  $x$  and  $y$  satisfy three of the four equations below but not the fourth. Find  $x^{-1}y^2$  as a fraction in reduced form

$$x + y = 78$$

$$x - y = 62$$

$$xy = 432$$

$$\frac{x}{y} = 12$$

ANSWER \_\_\_\_\_  $\frac{1}{2}$

3) If one is the “loneliest number” because it has the least number of positive integral factors, what is the “least loneliest number” (the number with the greatest number of positive integral factors) that is a positive three-digit integer?

ANSWER \_\_\_\_\_ 840

4) Twenty-five identical cylindrical shaped glasses with radii of three inches and heights of eight inches are filled with liquid to within one inch of the top. If each glass contains five ice cubes that are .8 inches on each edge, find the total number of cubic inches of liquid in the glasses. (Use  $\frac{22}{7}$  for  $\pi$ )

ANSWER \_\_\_\_\_ 4886

5) If the product  $1,001,001,001 \times 999 \times 1,000,000,000,001$  is computed, how many 9's are in the product?

ANSWER \_\_\_\_\_ 24

- 6) A group of 75 students went to Washington D.C.
- \* 20 students visited the Washington Monument
  - \* 35 students visited the White House
  - \* 15 students visited the Lincoln Memorial
  - \* 10 students visited the White House and Lincoln Memorial
  - \* 8 students visited the White House and Washington Monument
  - \* 6 students visited the Washington Monument and Lincoln Memorial
  - \* 4 students visited all three sites

How many students visited none of these sites?

ANSWER \_\_\_\_\_ 25

- 7) Let A = The sum of all positive two-digit numbers that are multiples of 8.  
Let B = The sum of all positive two-digit numbers that are multiples of 10.  
Let C = The sum of all positive two-digit perfect squares.

Find  $A + B + C$

ANSWER \_\_\_\_\_ 1337

- 8) A four-digit number is formed using the digits 3, 4, 5, and 6. Each digit is used only once. What is the probability that the results will be divisible by 36?

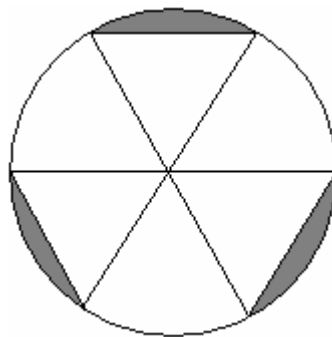
ANSWER \_\_\_\_\_  $\frac{1}{4}$  or .25 or 25%

9) In the following sequence, each term starting with the third is the sum of the two preceding terms. Find the second term.

5, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, 9

ANSWER \_\_\_\_\_ -2

10) Circle O has a diameter of 20cm and triangles inside the circle are equilateral. Using 3.14 as an approximation for  $\pi$ , what is the portion of the circle that is shaded (to the nearest whole percent)?



ANSWER \_\_\_\_\_ 9

11) The sequence 1, 2, 3, 4, 6, 7, 8, 9, 11, .....is obtained by omitting every fifth natural number. What is the sum of the first 100 terms of this sequence?

ANSWER \_\_\_\_\_ 6250

12) If  $d$  is a digit and  $a$  is an integer and  $[3(a + 134)]^2 = 1d8921$ , find the sum of  $a$  and  $d$ .

ANSWER \_\_\_\_\_ 9

13) When three standard dice are rolled what is the probability that there is at most one 5? Express your answer as a common fraction.

ANSWER \_\_\_\_\_  $\frac{25}{27}$

14) Express the product of  $1100101_2$  and  $3012_4$  as a base 8 number.

ANSWER \_\_\_\_\_  $47036$  or  $47036_8$



15) If  $(x,y,z)$  is the solution to the following system of equations, find  $(xyz)^2$ .

$$x + y - 2z = -10$$

$$2x + y - z = -2$$

$$3x + 4y + 2z = 7$$

ANSWER \_\_\_\_\_ 2025

16) How many integers are there that satisfy  $|2x - 40| < 1000$

ANSWER \_\_\_\_\_ 999

17) Within a standard  $8 \times 8$  checkerboard there are 204 squares of varying sizes can be identified (some  $1 \times 1$ , some  $2 \times 2$ , etc.). In how many of these squares is the area of the black unit squares greater than the area of the red unit squares?

ANSWER: \_\_\_\_\_ 60

18) How many positive three-digit numbers have at least one 7 as a digit?

ANSWER \_\_\_\_\_ 252

19) When the digits 2 through 9 are placed in the blanks below, with each digit used only once, what is the maximum value of the expression? (Some digits are already placed)

$$\underline{9} \_ \_ \_ + ( \underline{8} \_ \times \underline{7} \_ )$$

ANSWER \_\_\_\_\_ 15,932

20) A palindromic prime number is a two-digit prime number such that the number formed when the digits are reversed is also prime. What is the sum of all two-digit palindromic primes?

ANSWER \_\_\_\_\_ 429